Welcome to college and to the Mathematics Department.

You will have a long break this summer and may well find that you get rather rusty at some of the maths skills which you spent so long learning at school.

This booklet contains some of those key ideas from GCSE which will help you to make a good start on the A-level course. Please work through this booklet over the summer to keep your skills up to speed.

Read through the examples for each topic and have a go at the questions in each bold section. Please make a good attempt at every question - we'd rather it was wrong than blank as it helps us to see where you may need some help! It's fine to look things up in your old books, or look at websites like BBC GCSE Bitesize to get some help if you need it.

Please set out all of your working carefully and don't use a calculator for any of these questions.

Hand in your completed booklet to your teacher on your first maths lesson - this may be your first day in college so make sure you bring it with you!

Preparing for lessons in September - please bring:

- A4 file paper (lined not squared is preferable)
- A ring binder folder with some file dividers
- Pens and pencils
- Highlighter pens
- This booklet to hand in!
The following words or phrases are commonly used in A-level maths - how many do you recognise? Please jot down a brief definition of each term - look them up if you're not sure...

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<td>The Subject of an equation</td>
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Fractions

Rules:
You can add or subtract fractions if they have a common denominator.

Multiply by multiplying the numerators and multiplying the denominators.

Dividing by a fraction is the same as multiplying by the reciprocal (turn it over!)

Don’t forget to cancel fractions into their simplest possible form.

Examples:

\[
\frac{1}{6} + \frac{2}{7} = \frac{7}{42} + \frac{12}{42} = \frac{19}{42}
\]

\[
\frac{1}{5} \times \frac{10}{13} = \frac{10}{65} = \frac{2}{13}
\]

\[
\frac{7}{8} - \frac{1}{4} = \frac{7}{8} - \frac{2}{8} = \frac{5}{8}
\]

\[
\frac{3}{10} \div \frac{2}{5} = \frac{3}{10} \times \frac{5}{2} = \frac{15}{20} = \frac{3}{4}
\]

Try the following:

\[
\frac{1}{3} + \frac{1}{4} =
\]

\[
\frac{5}{8} - \frac{1}{2} =
\]

\[
\frac{1}{8} \times \frac{4}{7} =
\]

\[
\frac{4}{7} \div \frac{2}{3} =
\]

\[
\frac{2}{5} \div 2 =
\]
Try the following algebraic fractions using the same rules. Simplify your final fractions as far as possible:

\[
\frac{1}{a} + \frac{1}{b} = \\
\frac{3}{x^2} - \frac{1}{x} = \\
\frac{2}{x^3} \times \frac{x^2}{8} = \\
\frac{1}{x} \div \frac{1}{y} = \\
\frac{a}{b} - \frac{a}{b + 1} = \\
\frac{1}{x + 1} + \frac{1}{x - 1} =
\]
Substitution: If $a = 2, b = -3$ and $c = 5$ find the values of the following expressions:

\[ a^2 + b^2 = \]
\[ 3ab - 2bc = \]
\[ (2a - b)(b + c) = \]
\[ 2c^2 - abc = \]

Expanding & Simplifying Examples

2(a + b) - 3(a - 2b) = 2a + 2b - 3a + 6b = -a + 8b

5 - 3(a - 2) = 5 - 3a + 6 = 11 - 3a

\[ (2a + 3b)(a - 2b) = 2a^2 - 4ab + 3ab - 6b^2 = 2a^2 - ab - 6b^2 \]

Try the following:

3(a + 10) + 2(a - 3) =

12 - 7(x + 3) =

(3x - y)(2x + 4y) =

(2a - 3)(2a + 3) =

(2x - 3)(x - 2) =
Algebra – Solving and Rearranging Equations

Examples:

\[3x + 7 = 19\]
\[\Rightarrow 3x = 12\]
\[\Rightarrow x = 4\]

\[D = b^2 - 4ac\]
\[\Rightarrow D + 4ac = b^2\]
\[\Rightarrow 4ac = b^2 - D\]
\[\Rightarrow c = \frac{b^2 - D}{4a}\]

Solve to find the value of \(a:\)

\[7a - 2 = 13 - 2a\]
\[\Rightarrow 7a = 15 - 2a\]
\[\Rightarrow 9a = 15\]
\[\Rightarrow a = \frac{15}{9} = \frac{5}{3}\]

Notice that the working is set out with one line below another.

Don’t write things like this:

\[3x + 7 = 19 = 3x = 12 = 4\]

You may finish in the right place but this lengthy list of = signs doesn’t make sense!

You don’t need to say what you’re doing each time – this is just here to remind you!

Rearrange to make \(c\) the subject:

\[\frac{a}{2} + 10 = 3a + 2\]

We prefer fractions – don’t change to decimals

Solve the following:

\[4x - 11 = 5\]
\[5 - 7z = -9\]

\[6 + 4(y - 1) = 12y + 2\]
\[5a - 3 = 11 - 2a\]

\[(x - 3)(x - 2) = (x + 1)(x - 5)\]
Rearrange the following formulae:

Make $y$ the subject:

$$3x - y = 5$$
$$3y - 6x - 4 = 0$$

Make $r$ the subject:

$$S = 4\pi r^2$$
$$x = m - 3r$$

Make $x$ the subject:

$$y = 3\sqrt{x + 2}$$
$$y(x + 2) = 4$$

Make $a$ the subject:

$$b = \frac{a^2}{2}$$
$$b = \frac{\sqrt{a + 2}}{c}$$
**Algebra – Factorising**

**Examples:**

Look for common factors (numbers or letters) to take out first...

\[ 4a - 8b = 4(a - 2b) \quad x^2 - 5x = x(x - 5) \quad 6yz - 9z^2 = 3z(2y - 3z) \]

Quadratic expressions - look for numbers which multiply to make the constant term and add up to give the \( x \) term.

\[ x^2 - 5x + 6 = (x - 2)(x - 3) \]

\[ a^2 + 6a - 27 = (a + 9)(a - 3) \]

Sometimes do both...

\[ 2x^3 - 14x^2 + 24x = 2x(x^2 - 7x + 12) = 2x(x - 3)(x - 4) \]

**Factorise the following:**

\[ 25a + 10b = \]

\[ 12ab - 8bc = \]

\[ 3x^3 + 9x^2 = \]

\[ a^2 + 7a + 6 = \]

\[ x^2 + x - 12 = \]

\[ b^2 - 7b - 30 = \]

\[ y^2 - 11y + 30 = \]

\[ 3a^2 - 3a - 60 = \]

**Pythagoras**

\[ x^2 = 5^2 + 12^2 \]

\[ x^2 = 25 + 144 \]

\[ x^2 = 169 \]

\[ x = 13 \]

**Find the value of \( x \)**

\[ x \]

[Diagram of a triangle with sides 5cm, 12cm, 10cm, and 6cm, with \( x \) as the hypotenuse, calculating \( x \) as 13 cm]
### Surds

**Rules:**
\[
\sqrt{a} \times \sqrt{b} = \sqrt{ab}
\]
\[
\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}
\]

**Examples**
\[
\sqrt{6} \times \sqrt{10} = \sqrt{60}
\]
\[
3(4 - \sqrt{2}) = 12 - 3\sqrt{2}
\]
\[
\frac{\sqrt{75}}{\sqrt{3}} = \frac{\sqrt{75}}{3} = \sqrt{25} = 5
\]
\[
2\sqrt{6} \times \sqrt{6} = 2\sqrt{36} = 2 \times 6 = 12
\]

### Try the following:
\[
\sqrt{5} \times \sqrt{2} =
\]
\[
3\sqrt{2} \times \sqrt{8} =
\]
\[
\frac{\sqrt{6}}{\sqrt{3}} =
\]
\[
\frac{5\sqrt{24}}{\sqrt{6}} =
\]

### Simplifying surds - can be done by writing surds as a product where one of the numbers is a perfect square

**Examples:**
\[
\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}
\]
\[
\sqrt{27} = \sqrt{9 \times 3} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}
\]
\[
\frac{\sqrt{18}}{6} = \frac{\sqrt{9 \times 2}}{6} = \frac{\sqrt{9} \times \sqrt{2}}{6} = \frac{3\sqrt{2}}{6} = \frac{\sqrt{2}}{2}
\]

### Simplify the following:
\[
\sqrt{8} =
\]
\[
\sqrt{50} =
\]
\[
\sqrt{12} =
\]
\[
\sqrt{75} =
\]
\[
\sqrt{288} =
\]
\[
\sqrt{98} =
\]
\[
\sqrt{40} =
\]
\[
\sqrt{48} =
\]
**Extension:** (Optional – have a go at this if you found the other sections straightforward. If you’re taking Further Maths you should definitely try this!)

*Without using a calculator*, decide whether the following are true or false and explain your reasoning:

\[ 5\sqrt{6} > 6\sqrt{5} \]

\[ \sqrt{32} + \sqrt{72} = \sqrt{200} \]

\[ \sqrt{45} - \sqrt{20} = \sqrt{25} \]

The diagram shows a triangle in a semi-circle. Work out the perimeter of the semi-circle giving your answer as an exact form involving \sqrt{5} and \pi. (no calculator / decimals)